agrees with known results for a spherically blunted region [4, 5]. The opposite effect occurs on the conical part of the body for $s/r > 1.8$. Here, allowing for physicochemical processes results in an increase in the coefficient of friction. The nonmonotonic variation of C_f on the conical part of the body comes about because for equilibrium air the singularity arising at the point where the sphere contacts the cone has a much stronger effect than in the case $\gamma = 1.4$, $Pr = 0.72$, $\mu \sim \sqrt{T}$. The change in the departure of the shock wave and in the pressure on the surface of the body also become appreciably nonmonotonic [3]. We note that for hypersonic flow around bodies with a surface of continuous curvature the coefficient of friction on the corresponding part decreases monotonically [6, 7].

In Fig. 2 we show profiles of the velocity component u tangential to the surface of the body. It can be seen that on different parts of the surface of the body, allowing for physicochemical processes can cause either an increase or a decrease in the velocity gradient in the region near the wall. These results demonstrate the need to allow for physicochemical processes when determining friction stresses on the surfaces of blunt bodies in hypersonic flows.

NOTATION

s, distance measured from front critical point along surface of body; n, distance along normal directed away from surface of body; C_f, coefficient of friction; u, component of velocity tangential to surface of body; V_∞, ρ_∞, velocity and density of unperturbed flow; r, radius of spherically blunted portion; T_W, surface temperature of body; γ, ratio of specific heats; Pr, Prandtl number; μ, viscosity.

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HEAT LIBERATION FROM A SPHERE MOVING IN A VISCOUS LIQUID

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The method of [1] is used to solve the problem of heat liberation from a sphere moving in a viscous liquid, where the velocity field is given by the Stokes solution [2].

1. Method of Solution. A method proposed previously [1] makes it possible to determine the nonstationary temperature gradient on the boundary of a semiinfinite one-dimensional region without previous determination of the temperature field. In the present section we will describe the application of this method to nonstationary problems for a two-dimensional region.

We will consider the simplest case — the process of heating a semiinfinite lamina from its face:

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$$\left(\frac{\partial}{\partial t}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)T=0, \tag{1}$$

$$0\leqslant x<\infty,\ -\infty<y<\infty,\ 0<t<\infty,$$
$$T|_{x=0}=T_0(y,\ t);\ T|_{x=\infty}=0;\ T|_{t=0}=0. \tag{2}$$

It is required that the temperature gradient in the x direction be found at the end of the lamina, i.e., $(dT/dx)_{x=0}$.

We write Eq. (1) in the form

$$\left(\sqrt{\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}}-\frac{\partial}{\partial x}\right)\left(\sqrt{\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}}+\frac{\partial}{\partial x}\right)T=0. \tag{3}$$

Here the square root operator is defined such that

$$\sqrt{\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}}\cdot\sqrt{\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}}f(x,\ y,\ t)=\left(\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}\right)f(x,\ y,\ t) \tag{4}$$

for an arbitrary function f.

It can be directly verified that the property of Eq. (4) is fulfilled if the operator is defined by a formal expansion of the root in a binomial series:

$$\sqrt{\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}}=\sum_{n=0}^{\infty}(-1)^n\binom{\frac{1}{2}}{n}\frac{\partial^{\frac{1}{2}-n}}{\partial t^{\frac{1}{2}-n}}\frac{\partial^{2n}}{\partial y^{2n}}, \tag{5}$$

where

$$\frac{\partial^{\nu}f(t)}{\partial t^{\nu}}=\frac{1}{\Gamma(1-\nu)}\frac{d}{dt}\int_0^t(t-z)^{-\nu}f(z)\,dz,\ \nu<1.$$

It develops that the solution of the equation

$$\left(\sqrt{\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}}+\frac{\partial}{\partial x}\right)T=0, \tag{6}$$

formed by the right-hand factor in Eq. (3) can satisfy all conditions of Eq. (2), in a manner analogous to [1]. The right-hand factor gives solutions finite as $x\to\infty$, while the left-hand term gives solutions finite as $x\to-\infty$. Therefore, writing Eq. (6) at x = 0, we obtain the desired solution in the form

$$-\left.\frac{\partial T}{\partial x}\right|_{x=0}=\sqrt{\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}}\,T_0(y,\ t), \tag{7}$$

where the square root operator is defined by Eq. (5).

The solution of Eq. (7) coincides with the solution obtained by other methods, for example, with the aid of integral transforms.

In similar manner, for a thermal-conductivity problem with variable coefficients, given the conditions of Eq. (2), division of the operator into two factors may be performed in the form

$$\left[\frac{\partial}{\partial t}-\alpha(x,\ y,\ t)\frac{\partial^2}{\partial x^2}-\beta(x,\ y,\ t)\frac{\partial^2}{\partial y^2}-\cdots\right]T$$
$$=\left[\sum_{n=0}^{\infty}\sum_{m=0}^{n}b_{mn}(x,\ y,\ t)\frac{\partial^{\frac{1-n}{2}}}{\partial t^{\frac{1-n}{2}}}\frac{\partial^m}{\partial y^m}-\alpha^{1/2}(x,\ y,\ t)\frac{\partial}{\partial x}\right]$$
$$\times\left[\sum_{n=0}^{\infty}\sum_{m=0}^{n}a_{mn}(x,\ y,\ t)\frac{\partial^{\frac{1-n}{2}}}{\partial t^{\frac{1-n}{2}}}\frac{\partial^m}{\partial y^m}+\alpha^{1/2}(x,\ y,\ t)\frac{\partial}{\partial x}\right]T=0, \tag{8}$$

where the functions a_{mn}, b_{mn} can be found from a system of recurrent relationships in analogy to [1]. Writing the equation formed by the right-hand factor of Eq. (8) at x = 0, we obtain a solution of the problem in the form of a series in fractional-order derivatives of the given function $T_0(y, t)$.

By applying to the system of equations (8)-(12) a Fourier transform in y and a Laplace transform in t, and performing averaging in the integrals, it can be shown that the validity of the method is determined by the absolute convergence, uniform with respect to x, of the series in Eq. (8). General conditions for convergence of the two-dimensional problem were not obtained.

The applicability of the method to problems for which convergence has not been established rests on the following facts.

1. "Practical convergence" of the calculated series.

2. Validity of the final result for all problems which could be verified by other methods.

3. The validity of the method for the one-dimensional case, where α, β, ... in Eq. (8) are analytic functions for all x, t, and the derivatives $d^n\alpha/dx^n$ do not increase "too rapidly" as $n \to \infty$ [3].

Since the double series in Eq. (8) have a triangular coefficient matrix, the division of Eq. (8) can be performed more economically by use of single series in fractional-order derivatives. To do this we introduce the notation

$$\sum_{m=0}^{n} a_{mn} \frac{\partial^m}{\partial y^m} = L_n, \quad \sum_{m=0}^{n} b_{mn} \frac{\partial^m}{\partial y^m} = M_n. \tag{9}$$

The solution process will be described in detail below for a concrete example.

2. Nonstationary Heat Liberation from a Sphere Moving in a Viscous Liquid. In an infinite volume of viscous liquid at temperature T = 0 a sphere of radius R moves in uniform rectilinear motion at velocity u. Beginning at time t = 0 the sphere surface is heated by a known law $T = T_s(\vartheta, t)$. In a spherical coordinate system attached to the sphere, heat transfer into the liquid will be described by the equation

$$\left\{\frac{\partial}{\partial\tau} - \frac{\partial^2}{\partial\rho^2} - \left[\frac{2}{\rho} - \mathrm{Pe}\cos\vartheta\left(1 - \frac{3}{2\rho} + \frac{1}{2\rho^3}\right)\right]\frac{\partial}{\partial\rho} - \frac{1}{\rho^2}\frac{\partial^2}{\partial\vartheta^2} - \left[\frac{\operatorname{ctg}\vartheta}{\rho^2} + \mathrm{Pe}\frac{\sin\vartheta}{\rho}\left(1 - \frac{3}{4\rho} - \frac{1}{4\rho^3}\right)\right]\frac{\partial}{\partial\vartheta}\right\}T = 0,$$

$$\rho \geqslant 1, \ \tau > 0, \ -\infty < \vartheta < \infty;$$

$$T|_{\rho=1} = T_s(\vartheta, \tau); \ T|_{\rho=\infty} = 0; \ T|_{\tau=0} = 0;$$

$$\rho = r/R, \ \tau = at/R^2, \ \mathrm{Pe} = uR/a. \tag{10}$$

It is required that the radial temperature gradient at the sphere surface $q_s = (dT/d\rho)_{\rho=1}$ be found.

We write the operator of Eq. (10) in the form of two cofactors, each of which depends only on the first derivative with respect to ρ [1, 3]:

$$\left[\sum_{m=0}^{\infty} M_m\left(\rho, \vartheta, \frac{\partial^k}{\partial\vartheta^k}\right)\frac{\partial^{\frac{1-m}{2}}}{\partial\tau^{\frac{1-m}{2}}} - \frac{\partial}{\partial\rho}\right]\left[\sum_{n=0}^{\infty} L_n\left(\rho, \vartheta, \frac{\partial^k}{\partial\vartheta^k}\right)\frac{\partial^{\frac{1-n}{2}}}{\partial\tau^{\frac{1-n}{2}}} + \frac{\partial}{\partial\rho}\right]T = 0. \tag{11}$$

We will now define the operators M_m, L_n. Multiplying the expressions in Eq. (11) and considering that $(d^\nu/d\tau^\nu)(d^\mu/d\tau^\mu) = d^{\nu+\mu}/d\tau^{\nu+\mu}$, $\nu + \mu \leq 1$, we compare the result with Eq. (10). Equating functions of derivatives of one and the same order in τ, we obtain a system of recurrent relationships which define M_m, L_n:

$$\frac{\partial^{1/2}}{\partial\tau^{1/2}}: M_1 + L_1 = 0;$$

$$\frac{\partial}{\partial\rho}: M_1 - L_1 = -\frac{2}{\rho} + \mathrm{Pe}\cos\vartheta\left(1 - \frac{3}{2\rho} + \frac{1}{2\rho^3}\right);$$

$$\frac{\partial^0}{\partial\tau^0}: M_2 + M_1L_1 + L_2 - \frac{\partial L_1}{\partial\rho} = -\frac{1}{\rho^2}\frac{\partial^2}{\partial\vartheta^2} - \left[\frac{\operatorname{ctg}\vartheta}{\rho^2} + \mathrm{Pe}\frac{\sin\vartheta}{\rho}\left(1 - \frac{3}{4\rho} - \frac{1}{4\rho^3}\right)\right]\frac{\partial}{\partial\vartheta};$$

$$\frac{\partial}{\partial\rho}\frac{\partial^{-1/2}}{\partial\tau^{-1/2}}: M_2 - L_2 = 0;$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\frac{\partial^{-n/2}}{\partial\tau^{-n/2}}: \sum_{p=0}^{n+2} M_{n+2-p}L_p - \frac{\partial L_{n+1}}{\partial\rho} = 0;$$

$$\frac{\partial}{\partial\rho}\frac{\partial^{-\frac{n+1}{2}}}{\partial\tau^{-\frac{n+1}{2}}}: M_{n+2} - L_{n+2} = 0, \; n \geqslant 1; \tag{12}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

Performing the operations of Eq. (12), it must be remembered that the operators obey rules stemming from the properties of differentiation

$$\frac{\partial}{\partial\vartheta}L_n = L_n\frac{\partial}{\partial\vartheta} + \frac{\partial L_n}{\partial\vartheta}.$$

In accordance with the explanation of Section 1, we write the equation formed by the right-hand operator in Eq. (11) at $\rho = 1$, which gives the desired solution:

$$\begin{aligned}
-q_s = {} & \frac{\partial^{1/2}T_s}{\partial\tau^{1/2}} + T_s - \left(\frac{\operatorname{ctg}\vartheta}{2}\frac{\partial}{\partial\vartheta} + \frac{1}{2}\frac{\partial^2}{\partial\vartheta^2}\right)\frac{\partial^{-1/2}T_s}{\partial\tau^{-1/2}} \\
& - \left[\frac{3}{8}\mathrm{Pe}\cos\vartheta + \left(\frac{3}{8}\mathrm{Pe}\sin\vartheta - \frac{\operatorname{ctg}\vartheta}{2}\right)\frac{\partial}{\partial\vartheta} - \frac{1}{2}\frac{\partial^2}{\partial\vartheta^2}\right]\frac{\partial^{-1}T_s}{\partial\tau^{-1}} \\
& + \left[\frac{15}{16}\mathrm{Pe}\cos\vartheta + \left(\frac{15}{16}\mathrm{Pe}\sin\vartheta - \frac{3}{4}\operatorname{ctg}\vartheta - \frac{1}{8}\frac{\cos\vartheta}{\sin^3\vartheta}\right)\frac{\partial}{\partial\vartheta}\right. \\
& \left. + \left(\frac{\operatorname{ctg}^2\vartheta}{8} - \frac{5}{8}\right)\frac{\partial^2}{\partial\vartheta^2} - \frac{\operatorname{ctg}\vartheta}{4}\frac{\partial^3}{\partial\vartheta^3} - \frac{1}{8}\frac{\partial^4}{\partial\vartheta^4}\right]\frac{\partial^{-3/2}T_s}{\partial\tau^{-3/2}} \\
& - \left[\frac{33}{16}\mathrm{Pe}\cos\vartheta + \left(\frac{39}{16}\mathrm{Pe}\sin\vartheta - \frac{3}{16}\frac{\mathrm{Pe}}{\sin\vartheta} - \frac{3}{2}\operatorname{ctg}\vartheta\right.\right. \\
& \left. - \frac{1}{2}\frac{\cos\vartheta}{\sin^3\vartheta}\right)\frac{\partial}{\partial\vartheta} + \left(\frac{9}{8}\mathrm{Pe}\cos\vartheta - \frac{\operatorname{ctg}^2\vartheta}{2} + \frac{1}{\sin^2\vartheta} - \frac{3}{2}\right)\frac{\partial^2}{\partial\vartheta^2} \\
& \left. + \left(\frac{3}{8}\mathrm{Pe}\sin\vartheta - \operatorname{ctg}\vartheta\right)\frac{\partial^3}{\partial\vartheta^3} - \frac{1}{2}\frac{\partial^4}{\partial\vartheta^4}\right]\frac{\partial^{-2}T_s}{\partial\tau^{-2}} + \cdots .
\end{aligned} \tag{13}$$

The expression is useful for practical problems at $\tau < 1$.

It can be shown that at the points $\vartheta = 0, \pi$ a solution does not exist. In fact, for a function $T_S(\vartheta, \tau)$ infinitely differentiable with respect to ϑ, the singular terms standing before identical derivatives with respect to τ cancel each other. This may be verified by applying any operator L_n to $\cos m\vartheta$. For the case where $T_S(\tau)$ is independent of ϑ, from Eq. (12) we can obtain

$$\begin{aligned}
-q_s = {} & \frac{d^{1/2}T_s}{d\tau^{1/2}} + T_s - \frac{3}{8}\mathrm{Pe}\cos\vartheta\frac{d^{-1}T_s}{d\tau^{-1}} + \frac{15}{16}\mathrm{Pe}\cos\vartheta\frac{d^{-3/2}T_s}{d\tau^{-3/2}} - \frac{33}{16}\mathrm{Pe}\cos\vartheta\frac{d^{-2}T_s}{d\tau^{-2}} \\
& + \left(\frac{39}{8}\mathrm{Pe}\cos\vartheta + \frac{27}{128}\mathrm{Pe}^2\cos^2\vartheta + \frac{81}{128}\mathrm{Pe}^2\sin^2\vartheta\right)\frac{d^{-5/2}T_s}{d\tau^{-5/2}} + \cdots
\end{aligned} \tag{14}$$

We find the mean radial gradient by integrating Eq. (14) over the sphere surface:

$$-\bar{q}_s = \frac{d^{1/2}T_s}{d\tau^{1/2}} + T_s + \frac{63}{128}\mathrm{Pe}^2\frac{d^{-5/2}T_s}{d\tau^{-5/2}} + \cdots .$$

NOTATION

a, thermal diffusivity of liquid; a_{mn}, b_{mn}, functions of coordinates and time; R, sphere radius; u, velocity of motion; T, temperature; T_0, temperature of end of lamina; T_S, temperature of sphere surface;

x, y, Cartesian coordinates; r, ϑ, spherical coordinates; t, time; M_m, L_n, auxiliary operators; d^ν/dt^ν, fractional differentiation operator; f, arbitrary function of argument; z, auxiliary variable; ρ, dimensionless radial coordinate; τ, dimensionless time; q_s, dimensionless radial temperature gradient at sphere surface, Pe, Peclet numbers; μ, ν, exponentiation and differentiation indices; m, n, p, summing indices.

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